

Estimating Behavioral Agent-Based Models for Financial Markets through Machine Learning Surrogates

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Abstract

Traditional economic assumptions such as rational, representative agents and efficient market hypothesis failed to explain the macro-behavior of financial markets. On the other hand, agent-based approach proves high potentials in modeling bounded rational and heterogeneous micro-behaviors. This approach captures important stylized facts of financial markets. However, the high complexity of estimating agent-based models parameters precludes using these models in the forecasting process. This problem limits the applicability of agent-based models in decision making and policy formulation processes. Thereafter, this research aims at introducing a prospect for estimating agent-based models for financial markets through surrogate modeling approach. Surrogate models are considered as novel parameter estimation method in economics though it is a well-defined method in engineering. Few efforts have been spent to estimate parameters using surrogate models.

Key words:

Agent-Based Models - Financial Markets - Machine Learning Surrogates

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1. Introduction

Traditional economic models failed to capture different aspects of complex adaptive systems such as, heterogeneity, adaptation, learning and anticipation, and emergence. The recent financial crisis of 2008 is considered as a clear example of this failure. Agent-based modeling provides a promising approach to model heterogeneous, bloodedly rational agents. This approach is suitable to imitate complex adaptive systems. Agent-based modeling provides decision makers, researchers, and investors with test beds to investigate the effect of different regularity policies. This advantage is banned when model parameters are not configured to their counterparts in the target. When it comes to prediction, traditional modeling provides more reliable results. Thereby, decisions cannot be made unless agent-based models are calibrated with real data. This fact prohibits the practical application of agent-based models in business and in policy making process.

There are many constraints imposed on estimating agent-based models. First, the complexity of agent-based models precludes parameter estimation process. Second, agent-based modeling is a bottom-up approach, where the agents are the main building blocks of the model. Thereafter, the macro behavior of the model results from the interaction between the micro behavior of the agents. Estimating agent-based models using econometric techniques (macro variables) to estimate parameters of micro behavioral rules. Moreover, in many models it is very hard to determine the macro equation the model follows. Third, some advances in estimating agent-based models indirectly are achieved. However, this method can be applied to estimate very limited number of parameters for very simple models. Additionally, this method is based on local sensitivity as its computations are very costly.

The rest of the paper is organized as follows. In Section 2, a brief background of estimating agent-based models is provided. The rationale behind this research is clarified along with research questions are presented in Section 3. The methods proposed to conduct this research are stated in Section 4. Section 5 concludes the paper.

2. Background and Rationale

Many researchers started to tackle this problem. Contributions can be classified into three main categories as follows (Fagiolo et al., 2017).

2.1 Econometric estimation. A direct estimation of the agent-based model parameters is derived from a closed-form solution of the unconditional distribution of returns [c.f. Westerhoff and Dieci (2006) and Alfrano et al. (2005)]. However, many critiques face this method such as;

- i. Most agent-based models are complex and involves multi-agents with different micro behaviors. Thereafter, finding closed-form solution for such models is not possible.
- ii. This method requires pre-assumption of the likelihood, which can be totally different from the real likelihood generating the data.
- iii. Different slots of behavior are matched together.

2.2 Indirect estimation. Some advances in estimating agent-based models indirectly are achieved. An indirect estimation of agent-based models aims at solving the following problem [c.f. Winker and Gilli (2001), Alfarano et al. (2005)];

$$\theta^* = \arg \min(X^{\text{emp}}, X^{\text{sim}}, \theta), \quad (1)$$

where X^{emp} and X^{sim} represent the set of chosen moments observed empirically and their counterpart derived from the agent-based simulation, respectively. However, the critiques of this method can be summarized as follows.

- i. Analytical solution to this minimization problem does not always exist making numerical methods inevitable.
- ii. Moments selection is arbitrary and choosing different moments may lead to different parameter estimates.
- iii. Tuning the parameters so the model can represent some specific characteristics may come at the cost of miss replicating other important stylized facts.
- iv. This procedure is computationally expensive as the modeler needs to run sufficient number of Monte Carlo simulations at different parameter settings. For each setting, calculations of the distance between empirical and simulated moments are needed to decide on the best parameter instance that optimizes θ^* .
- v. Optimal parameter configurations might be off the grid, causing lots of evaluations in bad areas, and lots of costly evaluations.

3. Machine learning surrogates

A new trend of agent-based models' calibration, which proposes exploring parameter space by combining machine learning and intelligent iterative sampling [c.f. Lamperti (2017) and van der Hoog (2017)]. The optimization of (1) may be computationally expensive and analytically intractable. These problems can be tackled by replacing (1) with an iterative process that encompasses the creation, optimization, and updating of a fast and analytically tractable surrogate model. Advantages of surrogate models can be summarized as follows.

- i. Surrogate models can be constructed without prior knowledge of the target.
- ii. Surrogate models are based on logical statements and mathematical formulation (usually simple).
- iii. Surrogate models are usually computationally cheap but requires large amounts of data for the accuracy reasons.

As previously stated, agent-based models play significant role in simulating highly complex, dynamic and non-linear systems. However, the limited ability for estimating these models greatly reduce their applicability. This research is of crucial importance as it enhances the applicability of agent-based models for financial markets. The main objective is to find a general framework for estimating agent-based models. Agent-based models calibrated with real data can be used to forecast reliably the effect of different regulating policies. The research is conducted to address the following questions.

- i. How can we build a surrogate model for agent-based financial markets models?
- ii. Which machine learning system would more accurate surrogate model?

4. Methods

The primary research method for this study is computational modeling. There are mainly three approaches are needed to conduct this research;

- iii. Agent-based financial markets modeling
- iv. Surrogate (meta-modeling) approach
- v. Machine learning

Three procedures are needed to construct a surrogate (emulator) model for an agent-based model (van der Hoog, 2017). First, an agent-based financial market model is formulated and synthetic time series data is generated. Second, a machine learning system is built and trained on the synthetic data. Third, the trained machine leaning system is empirically validated using real financial time series data. At the end, the trained, empirically validated machine learning system can be applied to economic policy analysis.

The construction of efficient emulators of agent-based financial market models can be designed along two broad research themes:

Theme 1: Micro-emulation of the behaviour of individual agents.

Theme 2: Macro-emulation of an entire agent-based financial market simulation, using the multivariate time series data.

Henceforth, two types of data are needed in this research;

- i. Time series financial data (collected from real financial market such as, Yahoo.Finance)
- ii. Synthetic data (the output of agent-based simulation).

Principal Component Analysis (PCA) could be proposed for the estimation process. PCA is one of the most commonly unsupervised machine learning techniques. It is used for dimensional reduction especially when many variables were involved in the analysis. Its main benefit is that it can deal with multicollinearity. Accordingly, the proposed estimation algorithm is as follows;

Algorithm 1: Steps for ABM parameter estimation following PCA algorithm

1. Collect time-series data for a real financial market
2. Generate time-series by a simple agent-based model (such as (Selim, et al., 2015)) at different values for model parameters
3. Run PCA on the real time-series and the large number of generated time-series in (2)
4. Follow PCA to identify parameter values that explain most of the data variability

Selim, et al. (2015) introduced behavioral agent-based model under loss aversion. The model contained two types of agents; traders and price makers. The traders could trade in each step following fundamental or technical strategies, or they can abstain the market. The detailed model could be represented in the following.

A log-linear price impact function is followed to describe the market maker behavior (2002). The price settlement function measures the relation between the quantity ordered (demand/supply) and the price of the asset. Therefore, the log-price of the asset in period $t+1$ is given by;

$$p_{t+1} = p_t + a(w_t^c D_t^c + w_t^f D_t^f) + \alpha_t \quad (2)$$

Where a is a positive price settlement parameter, D_t^c and D_t^f are the orders submitted by chartists and fundamentalists; respectively, at time t and w_t^c and w_t^f are the weights of technical strategy and fundamental strategy; respectively, at time t . Noise terms α_t are added to represent random factors affecting the price settlement process. $\alpha_t, t = 1, 2, \dots, T$ are assumed to be IID normally distributed random variables with mean zero and constant standard deviation σ_α .

Chartists follow technical analysis to exploit the price changes (Murphy, 1999). Orders utilizing technical trading rules at time t can be presented by;

$$D_t^c = b(p_t - p_{t-1}) + \beta_t \quad (3)$$

where b is a positive reaction parameter that captures the strength of agents' sensitivity to price signals. In Eq. (3), the first term at the right-hand side depicts the deviation between current and previous price, which represents the exploitation of price changes. The second term represents additional random orders of technical trading rules, where $\beta_t, t = 1, 2, \dots, T$ are IID normally distributed random variables with mean zero and constant standard deviation σ_β .

On the other hand, fundamental analysis assumes that prices will return to their fundamental values in the long run (Graham & Dodd, 2009). Orders created by fundamental trading rules at time t can be presented as;

$$D_t^f = c(F_t - p_t) + \gamma_t \quad (4)$$

where c is a reaction parameter for the sensitivity of fundamentalists' excess demand to differences of the price from the underlying fundamental value. F_t are log-fundamental values (or simply fundamental values) (Day & Huang, 1990). γ_t is added to depict additional random orders of fundamental trading. $\gamma_t, t = 1, 2, \dots, T$ are IID normally distributed random variables with mean zero and constant standard deviation σ_γ .

The evolutionary switching behavior between trading strategies, proposed by Brock and Hommes (1998), illustrates how agents' beliefs are evolved over time. The evolving behavior is reflected in the weights $w_t = \{w_t^c, w_t^f, w_t^0\}$, where w_t^0 represents the fraction of inactive agents and w_t^c, w_t^f are as indicated in Eq. (2), thereafter, strategy weights add up to one. Weights are updated according to evolutionary fitness measure (or attractiveness of the trading rules) that could be presented as follows;

$$A_t^0 = 0 \quad (5)$$

$$A_t^c = (\exp(p_t) - \exp(p_{t-1}))D_{t-2}^c + mA_{t-1}^c \quad (6)$$

$$A_t^f = (\exp(p_t) - \exp(p_{t-1}))D_{t-2}^f + mA_{t-1}^f \quad (7)$$

where A_t^c, A_t^f , and A_t^0 are the fitness measures of using chartist strategy, fundamental strategy, and no-trade strategy, respectively. Inactive traders submit zero orders, so they receive zero attractiveness of making such decision. Fitness measure of the other two agents; the chartists and the fundamentalists, relies on two components. The first term of the right-hand sides of (6) and (7) is the performance of the trading strategy in most recent time. Note that, orders submitted in period $t - 2$ are implemented at the price stated in period $t - 1$.

The gains or losses depend on the price acknowledged in period t . The second term of the right-hand sides of (6) and (7) characterizes agents' memory, where $0 \leq m \leq 1$ is the memory parameter that evaluates the speed of recognizing present myopic profits. For $m = 0$, agent has no memory, while for $m = 1$ they calculate the fitness of the rule as a sum of all witnessed myopic profits.

Westerhoff (2008) suggests that agents symmetrically recognize gains and losses in terms of fitness. However, in this model a realistic behavioral bias is assumed, so that; chartists follow a value function of gains and losses to evaluate their strategies. The suggested value function indicates that, chartists recognize losses more than twice their perception of gains. We follow the Tversky and Kahneman (1991) and Benartzi and Thaler (1993) piecewise linear value function proposed by the prospect theory to apply the loss-aversion behavior. Consequently, the value of the fitness of technical strategy is provided by;

$$v_c = \begin{cases} A_t^c & \text{if } A_t^c \geq 0 \\ \lambda A_t^c & \text{if } A_t^c < 0 \end{cases} \quad (8)$$

where λ is the parameter of loss aversion that measures the relative sensitivity to gains and losses. Nevertheless, setting $\lambda = 1$ reduces the value function to; $v_c = A_t^c$. This situation represents loss-neutral chartists.

The market share of each trading strategy could be measured by the discrete choice model¹ (Manski and McFadden (1981)), as follows;

¹ A discrete choice model specifies probabilities $P(i|z, \theta)$ for each set of alternatives $\{i\}$ among which the decision maker can choose. The exogenous variables z describe observable attributes and characteristics of the decision maker and available alternatives to her/him. The parameters θ are to be estimated from the observed choices of a sample of decision makers. The

$$\begin{aligned}
 w_t^c &= \frac{\exp(rv_c)}{\exp(rA_t^0) + \exp(rv_c) + \exp(rA_t^f)} \\
 &= \frac{\exp(rv_c)}{1 + \exp(rv_c) + \exp(rA_t^f)} \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 w_t^f &= \frac{\exp(rA_t^f)}{\exp(rA_t^0) + \exp(rv_c) + \exp(rA_t^f)} \\
 &= \frac{\exp(rA_t^f)}{1 + \exp(rv_c) + \exp(rA_t^f)} \quad (10)
 \end{aligned}$$

$$\begin{aligned}
 w_t^0 &= \frac{\exp(rA_t^0)}{\exp(rA_t^0) + \exp(rv_c) + \exp(rA_t^f)} \\
 &= 1 - w_t^c - w_t^f \quad (11)
 \end{aligned}$$

The highest attractive strategy will be chosen by the more agents. The parameter r , in (9), (10), and (11) is called the intensity of choice and measures the sensitivity of mass of agents selecting the trading strategy with higher fitness measure. In such adaptive scheme, financial market prices and fractions of trading strategies will coevolve over time.

Ezzat (2021) estimates this model's parameters following indirect estimation method. However, this method could be used to estimate only few number of the model's parameters as follows,

$$\begin{aligned}
 &0.33ac > 0, \\
 &c < 6/a + 2b, \\
 &(12) \\
 &b < 3/a.
 \end{aligned}$$

choice probabilities are determined by the multinomial logit model as follows; $P(i|z, \theta) = \frac{\exp V_i(z, \theta)}{\sum_{j=1}^M \exp V_j(z, \theta)}$ where M is the number of available alternatives. And $V_i(z, \theta)$ is a summary statistic measuring the attractiveness of alternative i . It has the linear form of $V_i(z, \theta) = z_i \cdot \theta$, for $i = 1, 2, \dots, M$ (Manski & McFadden, 1981).

Accordingly, we propose taking the advantage of the computational power of the proposed PCA to estimate the rest of the model parameters, which are $\{m, r, \sigma_\alpha, \sigma_\beta, \sigma_\gamma, \lambda\}$. The real time-series returns will be included along with the generated time series at different values. The results will show the values of the parameters that could capture the most variability of the data set. The estimated values of the parameters could be used in the ABM model to predict the real financial market.

5. Conclusion

Agent-based financial markets performed very well in capturing complex features of real financial markets. This enables these models to be used as testbeds by decision makers, researchers, and investors. However, estimating the model parameters remains to be a dilemma. In this research, a prospect of using machine learning methods to estimate agent-based models is introduced.

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